

# Optimal Undervoltage Load Shedding in a Restructured Environment

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**Abstract-** Load shedding is a critical issue in power system especially under restructured electricity environment. Load shedding in reregulated power systems associated with security and reliability is investigated. This paper presents an efficient and reliable evolutionary based approach to reveal the optimal load shedding scheme considering voltage stability. System operators can predict the location and the amount of load shed to avoid a voltage collapse. For this purpose minimum eigen value of load flow Jacobian has been selected as proximity indicator. A computational algorithm for minimum amount of load shedding has been evaluated using Differential Evolution. Proposed methodology has been implemented on IEEE 14 bus system. The obtained results show the effectiveness of the proposed methodology in implementing the optimal load shedding to avoid voltage collapse under non-correction state.

**Keywords—** load shedding, voltage stability, proximity indicator, differential evolution.

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## 1. Introduction

A 'restructured environment' is one which ensures secure and economic operation of the power system involving a sure balance between generation and demand, continuity in service and stability of power system through social welfare maximization. However the restructuring of the power industry is facing new challenges, such as secure operation of the grid, congestion management, power quality, frequency and power regulation [1, 2]. Any disturbance in the power system like generator or line contingency or a sudden increase in load demand leads to insecure operation ultimately leading to voltage instability. Thus within restructured environment, voltage stability issues are becoming significant in the way we plan, operate and maintain the system. With the expanding scale of the power grid and development of power market, system operation is running to its limit. Voltage stability refers to the ability of a power system to maintain steady state voltages at all the load busses under normal operating condition and after occurrence of a disturbance. The phenomenon of voltage collapse on a transmission system is often caused by a low voltage initial profile, excessive demand, operation near the maximum power to be transmitted; generating facilities located too far from demand or insufficiency of reactive power compensation facilities. Voltage instability phenomenon arises when a disturbance, increase in load demand, or change in power system operational condition initiates an escalating and uncontrollable drop in voltage level [3]. At first a gradual voltage drop in one or several consumer regions may lead to increased reactive losses in the system and push transformer taps towards maximum values. Some generators or compensators can reach their limits of reactive power. Then voltage drops rapidly and it may drop so far as to cut off generating units and lines one after the other thus causing a complete collapse of the system. Thus a common limiting factor for power transmission is the risk of voltage instability in recent years. There are two well-known methods for maintaining voltage stability: preventive and corrective actions [4, 5].

Preventive actions are performed based on pre contingency state through applying required control strategies to provide a satisfactory margin. On the other hand corrective actions are performed when sever disturbance is imposed on the system and tries to return the system within its security margin. A power system might be in normal, alert, emergency, in extremis and restorative states. Load curtailment can be applied when the system is in the emergency state while load shedding is employed when system is in extremis state and is driven to collapse. The ultimate countermeasure to voltage collapse, load shedding is normally considered the last resort, when there are no other alternatives to stop an approaching voltage collapse. Load shedding (LS) is generally categorized in two well-known methodologies: under-frequency load shedding (UFLS) and under-voltage load shedding (UVLS). UFLS or UVLS is performed when the frequency or voltage falls below a specified threshold. Load shedding procedure cuts the particular amount of load in such a manner that a balance between generation and demand is achieved resulting in a widespread system blackout prevention. The main factors in load shedding are: location, amount, and time of load cut. On the other hand, to prevent post contingency problems, the location of the proposed buses for load shedding must be determined based upon the load importance, curtailment cost and the distance of the curtailed load to the contingency location. The phenomena of voltage collapse are complicated. An exact calculation to estimate voltage stability limits is difficult. A rapid indicator is used to identify on line risk of voltage instability. Thus load shed criterion may be based on some indicators whose magnitude indirectly reflects the stability margin and provides information for initialization of load shedding. The magnitude of this indicator is monitored and when there is an escalating fall in its value load shed is initiated. In this paper a new algorithm has been proposed considering the operating characteristics and stability inequality constraints for optimum load shedding at selected busses. The busses have been ranked based on sensitivity of proximity indicator [6, 7, 8] of the load flow Jacobean which is obtained using continuation power flow method. During emergency load shedding is required proximity indicator falls below a threshold value and load shedding is initialized at the load busses with higher sensitivities. Amount of load shed has been optimized using differential algorithm [3, 11].

## 2. Problem formulation

Load shedding is carried out at minimum number of busses. Operating and stability constraints decide the location and the upper limit of load shed. Busses are ranked in descending order of sensitivity. Once busses are selected for load shedding the following objective function is minimized at current operating condition.

$$J = \sum_{i=NLS} (LS_i) \quad (1)$$

Above objective function is optimized subject to following constraints under current operating condition as well as next predicted loading condition accounting load shed:

- (i) Power flow constraints:

$$\begin{aligned} P &= f(V, \delta) \\ Q &= g(V, \delta) \end{aligned} \quad (2)$$

- (ii) Inequality constraint on minimum eigen value proximity indicator of load flow Jacobian:

$$\begin{aligned} \tau_o &\geq \tau_{th} \\ \tau_p &\geq \tau_{th} \end{aligned} \quad (3)$$

- (iii) Active power generation under base case condition as well as at next operating condition accounting load shed:

$$P_{gk} \leq P_{gk}^0 \leq P_{gk}$$

$$\underline{P}_{gk} \leq P_{gk}^p \leq P_{gk}, \quad K=1, 2, \dots, NG \quad (4)$$

(iv) Reactive power generation constraint:

$$\begin{aligned} \underline{Q}_{gk} &\leq Q_{gk}^0 \leq Q_{gk} \\ \underline{Q}_{gk} &\leq Q_{gk}^p \leq Q_{gk} \\ K &= 1, 2, \dots, NG \end{aligned} \quad (5)$$

(v) In equality constraint on load bus voltage:

$$\begin{aligned} \underline{V}_i &\leq V_i^0 \leq V_i \\ \underline{V}_i &\leq V_i^p \leq V_i \\ i &= NG+1, \dots, NB \end{aligned} \quad (6)$$

(vi) Load shedding constraints:

$$\underline{ls}_i \leq ls_i \leq ls_i, \quad i \in NLS \quad (7)$$

Due to operating constraints there is a maximum limit to the load that can be shed at each bus (for example 80% of the initial load of the bus) to ensure a “minimum” service, and the load can only be shed in steps of for example 20, 40, 60 or 80% of the initial load. The method takes into account these constraints. Load shedding is performed at current loading condition and the constraints mentioned in equations (2)-(6) are ascertained by performing load flow solution at current operating condition (after load shedding) and predicted load condition (accounting load shed).

### 3. Differential Evolution: An overview [11]

Differential Evolution (DE) is a very simple population based, stochastic function minimizer and has been found very powerful to solve various natures of engineering problems[3]. DE optimizes the problem by iteratively trying to improve a candidate solution with regard to a given measure of quality. Thus optimization is achieved by sampling the objective function at multiple randomly chosen initial points. Preset parameter bounds define the region from which ‘M’ vectors in this initial population are chosen. DE generates new solution points in ‘D’ dimensional space that are perturbations of existing points. It perturbs vectors with the scaled difference of two randomly selected population vectors. To produce a mutated vector, DE adds the scaled, random vector difference to a third selected population vector (base vector). Further, DE also employs a uniform crossover to produce trial vector from target vector and mutated vector. The four fundamental steps are explained below:

**Step - (a) initialization:** initial population of size ‘M’ is generated as follows:

$$\begin{aligned} ls^0 &= [x_1^0, x_2^0, x_3^0, \dots, x_m^0] \\ x_i^0 &= [ls_{i,1}^0, ls_{i,2}^0, ls_{i,3}^0, \dots, ls_{i,NLS}^0]^T \end{aligned} \quad (8)$$

$ls_{ij}$  is the  $j$ th parameter of  $x_i$  vector and is obtained from uniform distribution as follows:

$$ls_{ij}^0 = \underline{ls}_j + (ls_j - \underline{ls}_j)rand_j \quad (9)$$

$\underline{ls}_j$  and  $ls_j$  are lower and upper bounds on variable  $ls_j$ .  $rand_j$  is a random digit in the range[0,1].

**Step – (b) mutation:** DE mutates and recombines the population to produce a population of ‘M’ trial vectors. Differential mutation adds a scaled, randomly sampled, vector difference to a third vector as follows:

For each target vector  $ls_i^{(k+1)}$  at generation M, an associated mutant vector  $\rho_i^{(k)} = ls_{1i}, ls_{2i}, ls_{3i}, \dots, ls_{ni}$  can usually be generated by using one of the following five strategies:

“DE/rand/1”

$$\underline{\rho}_i^{(k)} = ls_{r_1}^{(k)} + \alpha(ls_{r_2}^{(k)} - ls_{r_3}^{(k)}) \quad (10)$$

“DE/best/1”:

$$\underline{\rho}_i^{(k)} = ls_{base}^{(k)} + \alpha(ls_{r_1}^{(k)} - ls_{r_2}^{(k)}) \quad (11)$$

“DE/current to best/1”:

$$\underline{\rho}_i^{(k)} = ls_i^{(k)} + \alpha(ls_{base}^{(k)} - ls_i^{(k)}) + \alpha(ls_{r_1}^{(k)} - ls_{r_2}^{(k)}) \quad (12)$$

“DE/best/2”:

$$\underline{\rho}_i^{(k)} = ls_{base}^{(k)} + \alpha(ls_{r_1}^{(k)} - ls_{r_2}^{(k)}) + \alpha(ls_{r_3}^{(k)} - ls_{r_4}^{(k)}) \quad (13)$$

“DE/rand/2”:

$$\underline{\rho}_i^{(k)} = ls_{r_1}^{(k)} + \alpha(ls_{r_2}^{(k)} - ls_{r_3}^{(k)}) + \alpha(ls_{r_4}^{(k)} - ls_{r_5}^{(k)}) \quad (14)$$

is known as scale factor usually lies in the range [0, 1];  $ls_{base}^{(k)}$  is known as base vector  $\rho_i^{(k)}$  is a mutant vector;  $ls_{r_1}^{(k)}, ls_{r_2}^{(k)}, ls_{r_3}^{(k)}, ls_{r_4}^{(k)}$  and  $ls_{r_5}^{(k)}$  are five randomly selected vectors (r1? r2? r3? r4? r5). The base vector index ‘b’ may be determined in verity of ways. This may be a randomly chosen vector (base? r1? r2? r3? r4? r5?).

**Step-(c) crossover.** DE employs a uniform crossover strategy .crossover generates trial vectors  $t_i^{(k)}$  as follows:

$$t_{ij}^{(k)} = \begin{cases} \rho_{ij}^{(k)}, & \text{if } (rand_j \leq c_r \text{ or } j=j_{rand}) \\ ls_{ij}^{(k)}, & \text{otherwise} \end{cases} \quad (15)$$

$C_r$  is a crossover probability lies in the range [0,1].  $C_r$  is a user defined value which controls the number of parameter values which are copied from the mutant, If the random number  $rand_j$  is less than or equal to  $C_r$ , the trial parameter is adopted from the mutant  $\rho_i^{(k)}$ . further, the trial parameter with randomly chosen index,  $j_{rand}$  is taken from the mutant to ensure the trial vector does not duplicate the target vector  $ls_i^{(k)}$ . Otherwise the parameter is adopted from the target vector  $ls_i^{(k)}$ .

**Step-(d) selection:** objective function is evaluated for target vector and trial vector is selected if it provides better values of the function than target vector as follows:

$$lS_i^{(k+1)} = \begin{cases} t_i^{(k)}, & \text{if } [f(t_i^{(k)}) \leq f(lS_i^{(k)})] \\ lS_i^{(k)}, & \text{otherwise} \end{cases} \quad (16)$$

The process of mutation, crossover and selection is executed for all target index 'I' and a new population is created till the optimal solution is obtained. The procedure is terminated if a maximum number of generations (k) have been executed or no improvement in objective function is noticed in a pre-specified generation. In this paper DE/best/1/bin has been selected. The first term after DE i.e. 'best' specifies the way base vector difference contribute to the differential. Last term 'bin' denotes binomial distribution that result because of uniform crossover. Number of parameter donated by mutant vector closely follows binomial distribution. It is to be noted that best, target and difference vector indices are all different.

### 3.1 Bounce back technique for handling bounds on decision variables

Some of the variables may cross the lower or upper bounds a mutant vector  $\rho_i^{(k)}$  in executing differential as governed by relation (10)-(14). Bounce back mechanism is adopted to bring such decision variable within limit. The bounce-back method replaces element which has violated limit by the new element whose value lies between the base parameter values and the bound being involved. The following relations are used for violated mutant vector elements.

$$\rho_{ij}^{(k)} = \begin{cases} lS_{\text{base}j} + \text{rand.} \cdot (lS_j - lS_{\text{base}j}), & \text{if } (\rho_{ij}^{(k)} \leq lS_j) \\ lS_{\text{base}j} + \text{rand.} \cdot (lS_j - lS_{\text{base}j}), & \text{if } (\rho_{ij}^{(k)} > lS_j) \end{cases} \quad (17)$$

## 4. Problem solution

Implementation of Differential Evolution algorithm to solve formulated problem.

Step-1: data input; reactive power control variables and system parameters (resistance, reactance, and susceptance etc.).

Step-2: base case load flow solution is obtained using continuation power flow methodology.

Step-3: next interval load is predicted.

Step-4: obtain load flow solution for the predicted next interval load.

Step-5: obtain sensitivities for selection of most critical load bus.

Step-6: initialization; generate population of size 'M' for load shedding. Generated population is uniformly distributed in the range  $[0, \overline{lS}_i]$

$$x_i^{(0)} = [lS_i^{(0)}, lS_{i,2}^{(0)}, \dots, lS_{i,NLS}^{(0)}]^T, \quad i=1,2,\dots,M$$

Step-7: run power flow program for each vector of the population and monitor all line quality constraints (2)-(6). If a vector satisfies the constraints call it 's' (feasible). Otherwise, call it 'NF' (not-feasible).

Step-8: calculate objective function for the feasible vectors.

Step-9: based on the value of objective function, identify the best solution vector  $lS_{\text{best}}$ . This is selected as a base vector.

Step-10: set generation count  $k=1$ .

Step-11: select target vector  $i=1$ .

Step-12: select two vectors  $lS_{r1}$  and  $lS_{r2}$  such that  $\text{best} \neq r1 \neq r2$

Step-13: generate a mutated vector using relation (11).

Step-14: if any component of mutated vector i.e.  $\rho_i^{(k)}$  violates the bounds on decision variable  $lS_j$  then apply bounce back technique using relation (17) and bring the violated variables within limit.

Step-15: apply uniform crossover using relation (15) to get trial vector  $t_i^{(k)}$ . If the trial vector satisfy load shedding in equality constraint (7) call it 'F' otherwise, 'NF'.

Step-16: apply Lampinen's criteria to select  $t_i^{(k)}$  in new population or reject it to retain  $lS_i^{(k)}$  in new population.

Step-17: increase target vector  $i=i+1$ , if  $i \leq M$ , repeat from step-12. Otherwise increase generation count  $k=k+1$ .

Step-18: if  $k \leq k_{\text{max}}$  repeat from step-11. Otherwise stop.

The implementation described above is applied in sequence to solve formulated problem. Solution of the problem gives anticipatory optimum load shedding at critical load buses based on sensitivity

## 5. Results and discussions

The developed algorithms have been implemented for generating load shedding strategies on IEEE 14-bus test system [9]. For this purpose system has been stressed by uniform loading such that proximity indicator has been reduced to very small value and there is severe violation of bus voltages.

### 14-Bus system

This system consists of three generator buses and eleven load buses. The desired range of load bus voltage is 0.95 pu–1.05 pu. Table 1 shows system load, PV-bus voltages, load bus voltages, value of proximity indicator under simulated stressed condition and static voltage stability limit. Because of network overloading buses automatically gets switched into PQ buses after hitting the maximum limit of their reactive power generation.

**Table 1. Load flow solution for 14-bus test system under stressed condition**

Bus no.	Bus voltage(pu)		Bus load (pu)	
1	$V_1$	1.0929	$S_{d1}$	0.0000
2	$V_2$	1.0379	$S_{d2}$	0.5154
3	$V_3$	1.0323	$S_{d3}$	1.9699
4	$V_4$	1.0323	$S_{d4}$	0.2763
5	$V_5$	0.8389	$S_{d5}$	0.0000
6	$V_6$	0.9289	$S_{d6}$	0.9831
7	$V_7$	0.8389	$S_{d7}$	0.0000
8	$V_8$	0.9262	$S_{d8}$	0.1601
9	$V_9$	0.808	$S_{d9}$	0.6939
10	$V_{10}$	0.7994	$S_{d10}$	0.2195
11	$V_{11}$	0.8262	$S_{d11}$	0.0806
12	$V_{12}$	0.8412	$S_{d12}$	0.1292
13	$V_{13}$	0.8235	$S_{d13}$	0.3012
14	$V_{14}$	0.7804	$S_{d14}$	0.1681

Total load ( $S_{di}$ )=5.3923 pu, proximity indicator ( $\tau$ )=0.2228, static voltage stability limit =6.2380pu.

PV-bus no. 3 has crossed the limit and other PV buses operating very close to their limits. Because of switching from PV bus to PQ bus, the dimension of system Jacobian undergoes a change (increase in dimension). This change in dimension results a drastic change in proximity indicator (sudden decrease). It imposes a challenging task of setting threshold value in algorithm such that desired threshold value of indicator is achieved while considering the fact that these PQ buses will switch back to PV buses after load shedding

**Table 2. Load bus ranking based on sensitivity of eigen value indicator with respect to system load for 14-bus system**

Sr. no.	Load bus	Sensitivity
1	11	0.3938 <sup>a</sup>
2	14	0.3154 <sup>a</sup>
3	10	0.3011 <sup>a</sup>
4	13	0.2548 <sup>a</sup>
5	9	0.2135 <sup>a</sup>
6	12	0.1875
7	4	0.1853
8	8	0.1434
9	6	0.0532

**Table 3. Effect of DE parameters on optimization of objective function and number of iteration required for convergence for 14-bus test system.**

Case	$\alpha$	$C_r$	J	No. of iterations for convergence
1	0.90	0.55	0.4053	757
2	0.90	0.50	0.4129	930
3	0.90	0.45	0.4085	730
4	0.85	0.75	0.4098	943
5	0.85	0.70	0.4106	922
6	0.85	0.65	0.4141	795
7	0.80	0.65	0.4021	840
8	0.80	0.6	0.4051	820
9	0.80	0.55	0.4034	945
10	0.80	0.5	0.4018	934
<b>11</b>	<b>0.75</b>	<b>0.45</b>	<b>0.4009</b>	<b>435</b>
12	0.75	0.4	0.4039	831
13	0.75	0.65	0.4074	943
14	0.75	0.6	0.4026	965
15	0.75	0.55	0.4100	940
16	0.70	0.5	0.4043	660
17	0.70	0.45	0.4032	831
18	0.70	0.4	0.4014	759
19	0.70	0.45	0.4075	815
20	0.70	0.40	0.4025	705

**Table 4. Bus voltages and load on load bus after load shedding with and without optimization techniques for 14 bus test system**

Bus no.	Without optimization				After optimized load shedding	
	Base case		Best initial solution based load shedding		DE	
	Load(pu)	Voltage(pu)	Load(pu)	Voltage(pu)	Load(pu)	Voltage(pu)
1	0.0000	1.0929	0.0000	1.0929	0.0000	1.0928
2	0.5154	1.0379	0.5154	1.0379	0.5154	1.0380
3	1.9699	1.0323	1.9699	1.0323	1.9699	1.0325
4	0.2763	0.8722	0.2763	1.0224	0.2763	1.0042
5	0.0000	0.8389	0.0000	0.9843	0.0000	0.9629
6	0.9831	0.9289	0.9831	1.0134	0.9831	0.9965
7	0.0000	0.8389	0.0000	0.9843	0.0000	0.9634
8	0.1601	0.9262	0.1601	1.0058	0.1601	0.9905
9	0.6939	0.808	0.4435	0.9744	0.5687	0.9525
10	0.2195	0.7994	0.1651	0.9719	0.1830	0.9509
11	0.0806	0.8262	0.0797	0.9894	0.0746	0.9708
12	0.1292	0.8412	0.1292	1.0031	0.1292	0.984
13	0.3012	0.8235	0.1487	0.9972	0.1870	0.9763
14	0.1681	0.7804	0.1227	0.9691	0.1181	0.9502

## 6. Conclusions

The best under voltage load shedding scheme in power systems is one that is able to separate the least possible loads of the network in the shortest time by considering power system constraints. In this manner, the network is recovered against voltage reduction in addition to protecting the network stability. Thus the triad of UVLS principles are (i) amount of load shed, (ii) the timing of load shedding event and (iii) the location at which load is to be shed. The previous load shedding schemes do not have the above mentioned properties. In these methods, the load shedding process is time consuming and moreover, the unnecessary and extra loads might get separated from the network. This paper focused on load shedding scheme for providing voltage stability. This proposed a new algorithm considering operating characteristics and stability inequality constraints for optimum load shedding at selected load busses. A computational algorithm for minimum load shedding has been proposed using Differential Evolution. Ranking of the load busses has been done based on their sensitivity of minimum eigenvalue of load flow Jacobian obtained by power flow method. Load busses with large sensitivities have been selected for load shedding. Optimization of the amount of load shed is achieved by the proposed algorithm. Load shedding has been performed at current operating condition and constraints are ascertained by performing load flow after load shed and predicted load condition. Because of the use of suitable data base of contingencies in the proposed method, the presented algorithm has suitable functioning in different loading condition. The results of simulation on 14 bus system show that the proposed load shedding is optimal related to conventional load shedding method. So the presented load shedding method is an optimal and fast adaptive scheme to solve many traditional load shedding problems.

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